Goldstein 7.13.

$ct' = \gamma(ct - \vec{\beta}\cdot\vec{r})$, Introduce $x \equiv \vec{\beta}\cdot\vec{r}$.

$\Rightarrow c^2t'^2 = \gamma^2c^2t^2 + \gamma^2x^2 - 2\gamma^2 x ct.$

$$\vec{r}' = \vec{r} + \frac{(\vec{\beta}\cdot\vec{r})\vec{\beta}(\gamma-1)}{\beta^2} - \vec{\beta}\gamma ct.$$

$$\Rightarrow \vec{r}'^2 = r^2 + \frac{2x^2(\gamma-1)}{\beta^2} - \left[2x\gamma ct + 2x(\gamma-1)\gamma ct\right] + \frac{x^2(\gamma-1)^2}{\beta^2} + \beta^2\gamma^2c^2t^2.$$

$$= r^2 + \frac{2x^2(\gamma-1)}{\beta^2} - 2x\gamma^2 ct + \frac{x^2(\gamma-1)^2}{\beta^2} + \beta^2\gamma^2c^2t^2.$$

$$r'^2 - c^2t'^2 = r^2 + \frac{2x^2(\gamma-1)}{\beta^2} + \frac{x^2(\gamma-1)^2}{\beta^2} + \beta^2\gamma^2c^2t^2 - \gamma^2c^2t^2 - \gamma^2x^2.$$

$$\frac{x^2(\gamma-1)}{\beta^2}\left[2+(\gamma-1)\right] = \frac{x^2(\gamma^2-1)}{\beta^2} = \gamma^2x^2$$

$$(\beta^2-1)\gamma^2c^2t^2 = -c^2t^2.$$

$$\Rightarrow \boxed{r'^2 - c^2t'^2 = r^2 - c^2t^2}$$

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