

Gudstein 7.13.

$$ct' = \gamma(ct - \vec{\beta} \cdot \vec{r}), \quad \text{Introduce } x \equiv \vec{\beta} \cdot \vec{r}.$$

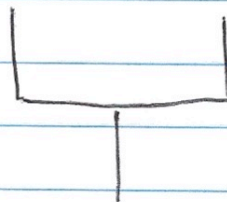
$$\Rightarrow c^2 t'^2 = \gamma^2 c^2 t^2 + \gamma^2 x^2 - 2\gamma x ct.$$

$$\vec{r}' = \vec{r} + \frac{(\vec{\beta} \cdot \vec{r})\vec{\beta}}{\beta^2}(ct-1) - \vec{\beta} \gamma ct.$$

$$\Rightarrow \vec{r}'^2 = r^2 + \frac{2x^2(ct-1)}{\beta^2} - [2x\gamma ct + 2x(ct-1)\gamma ct] + \frac{x^2(ct-1)^2}{\beta^2} + \beta^2 \gamma^2 c^2 t^2.$$

$$= r^2 + \frac{2x^2(ct-1)}{\beta^2} - 2x\gamma^2 ct + \frac{x^2(ct-1)^2}{\beta^2} + \beta^2 \gamma^2 c^2 t^2.$$

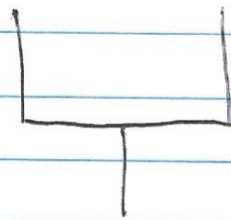
$$r'^2 - c^2 t'^2 = r^2 + \frac{2x^2(ct-1)}{\beta^2} + \frac{x^2(ct-1)^2}{\beta^2} + \beta^2 \gamma^2 c^2 t^2 - \gamma^2 c^2 t^2 - \gamma^2 x^2.$$



$$\frac{x^2(ct-1)[2+(ct-1)]}{\beta^2}$$

$$= \frac{x^2(\gamma^2 - 1)}{\beta^2}$$

$$= \gamma^2 x^2.$$



$$(\beta^2 - 1)\gamma^2 c^2 t^2 = -c^2 t^2.$$

$$\Rightarrow \boxed{r'^2 - c^2 t'^2 = r^2 - c^2 t^2}$$

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